Math 220, Discrete Mathematics, Spring 2017
Midterm 1 Practice Test

Instructions:

- **Please read each question carefully.**

- **No calculators, notes, books, or outside help of any kind are allowed to be used on this exam. Please turn cell phones off!**

- **Show all of your work and explain your answers clearly. In order to receive full credit your work must be complete, clear, and logical.**

- **Please cross out or fully erase any work that you do not want graded.**

1. Let a be an integer. Prove $a^2 - 2a + 5$ is even if and only if a is odd.

2. Prove
$$-1 + 2 - 3 + \cdots - (2n - 1) + 2n = n$$
   for all natural numbers n.

3. Let D_k be the set of *prime* divisors of k.

   (a) Find D_1 and D_{20}.

   (b) Find $|D_{10} \times D_{15}|$.

   (c) Find the power set of D_6.

4. Use the Euclidean Algorithm to find $\gcd(1320, 231)$

5. Construct a truth table to show that the statement $P \Rightarrow Q$ is equivalent to the statement $(\sim P) \vee Q$.

6. Suppose A, B, and C are sets. Explain whether the following are true or false.

   (a) $(A \backslash B) \cap C = (C \backslash B) \cap A$.

   (b) $(A \backslash B) \cup C = (C \backslash B) \cup A$.

7. Consider the statement: $\forall a \in \mathbb{Z}$ and $b \in \mathbb{N}$, $\exists c \in \mathbb{N}$ such that $ac > ab$.

   (a) Write down the negation of the statement.

   (b) Is the original statement true or false? Explain your answer.